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B.Sc- part-I (sub) Real Analysis 29-05-2020.
Problems based on continuity and differentiability.

Q.1 If $f(x) = x\sin x\frac{1}{x}$, for $x \neq 0$, and $f(0)=0$, show that $f(x)$ is continuous at $x=0$.

Proof: It is given that $f(0)=0$.

Now, $\lim_{h\to 0} f(0+h) = \lim_{h\to 0}(0+h)\sin\frac{1}{(0+h)} = \lim_{h\to 0} h\sin\frac{1}{h} = 0$

and $\lim_{h\to 0} f(0-h) = \lim_{h\to 0}(0-h)\sin\frac{1}{(0-h)} = \lim_{h\to 0} h\sin\frac{1}{h} = 0$.

So, $\lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(0-h) = f(0)$.

Hence, the given function is convergent at $x=0$.

Q.2. Given that $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0, x=0 \end{cases}$

Test the continuity of $f(x)$ at $x=0$,

Soln: We know that the given function $f(x)$ will be continuous at $x=0$, if

$\lim_{h\to 0} f(0+h) = f(0) = \lim_{h\to 0} f(0-h)$

Now, $\lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\sin(0+h)}{0+h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$

$f(0)=2$, by question.

$\lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin(0-h)}{0-h} = \lim_{h\to 0}\frac{-\sin h}{-h} = \lim_{h\to 0}\frac{\sin h}{h} = 1$

We find that $\lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(0-h) \neq f(0)$.

Hence, the given function is not cotinuous at $x=0$.

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Q. 3. Discuss the continuity at $x=0, 1, 2$ of the functions

$$f(x) = -x^2, \text{ for } x \le 0$$
$$f(x) = 5x-4, \text{ for } 0 < x \le 1$$
$$f(x) = 4x^2-3x, \text{ for } 1 < x < 2$$
$$f(x) = 3x+4, \text{ for } x \ge 2$$

Soln: To test the continuity of $f(x)$ at $x=0$

We have, $\lim_{h\to0} f(0+h) = \lim_{h\to0} \{5(0+h)-4\} = \lim_{h\to0} (5h-4) = -4$

$\lim_{h\to0} f(0-h) = \lim_{h\to0} \{-(-h)^2\} = \lim_{h\to0} (-h^2) = 0$ and $f(0) = 0$.

Since, $\lim_{h\to0} f(0+h) \neq \lim_{h\to0} f(0-h)$,

therefore, the function $f(x)$ is not continuous at $x=0$.

To test the continuity at $x=1$, We have

$\lim_{h\to0} f(1-h) = \lim_{h\to0} [5(1-h)-4] = 1.$

$\lim_{h\to0} f(1+h) = \lim_{h\to0} [4(1+h)^2 - 3(1+h)] = 4-3 = 1$

and $f(1) = 5.1-4 = 1$

Since, $\lim_{h\to0} f(1-h) = f(1) = \lim_{h\to0} f(1+h)$

therefore, the given function $f(x)$ is continuous at $x=1$.

To test the continuity at $x=2$, We have

$\lim_{h\to0} f(2-h) = \lim_{h\to0} [4(2-h)^2 - 3(2-h)] = 16-6 = 10$

$\lim_{h\to0} f(2+h) = \lim_{h\to0} [3(2+h)+4] = 6+4 = 10$

and $f(2) = 3.2+4 = 6+4 = 10.$

Since, $\lim_{h\to0} f(2-h) = f(2) = \lim_{h\to0} f(2+h)$

therefore, $f(x)$ is continuous at $x=2$

A.K.S.